

(2 ½ Hours)

[Total Marks: 75]

- N.B. 1) All questions are compulsory.
 2) Figures to the right indicate marks.
 3) Illustrations, in-depth answers and diagrams will be appreciated.
 4) Mixing of sub-questions is not allowed.

Q. 1 Attempt All(Each of 5Marks)**(15M)****(a) Multiple Choice Questions.**

- i) The set of all linear combinations of vectors $v_1, v_2, \dots, v_n$ is called the _____ of the vectors.
 a) Convex b) concave c) span d) combination.
- ii) Nullity of T is the dimension of _____ of T.
 a) Kernel b) Image c) Rank d) none of the above.
- iii) $|A - \lambda I| = 0$ is called _____ equation.
 a) Quadratic b) characteristics c) cubic d) Null.
- iv) In GF (2), $1+1+0+1 =$ _____.
 a) 0 b) 1 c) 3 d) 2.
- v) For any homogenous system _____ is a trivial solution.
 a) Zero b) non zero c) one d) none of the above.

**(b) Fill in the blanks.
(Spare, Unique, Unit, $\sqrt{45}$, Inner product)**

- i) A vector whose norm is one is called _____ vector.
- ii) A vector space together with inner product is called _____ space.
- iii) If most of the element of a matrix have zero value is called _____ matrix.
- iv) The absolute value of $3+6i =$ _____.
- v) Inverse of a matrix is _____.

(c) Define.

- i) Dot product.
- ii) Gatois field.
- iii) Eigen Value.
- iv) Orthogonal Complement.
- v) Dimension.

Q. 2 Attempt the following (Any THREE) (15M)

- (a) Find the Square root of $21 - 20i$, where $i = \sqrt{-1}$
- (b) Consider the following system of equation and find the nature of solution without solving it.

$$\begin{aligned} \text{i)} \quad & x_1 + x_2 = 4 \\ & 2x_1 + 2x_2 = 8 \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad & x_1 + x_2 = 3 \\ & x_1 - x_2 = -1 \end{aligned}$$

- (c) Solve the following system by backward substitution method
- $$\begin{aligned} x_1 - 3x_2 - 2x_3 &= 7 \\ 2x_2 + 4x_3 &= 4 \\ 10x_3 &= 20 \end{aligned}$$
- (d) Let W_1 and W_2 are two subspaces of V then prove that $W_1 \cap W_2$ is also a subspace of V where V is a vector space on $\mathbb{R}$.
- (e) Write a python Program for rotating a complex number $Z = 2+3i$ by 180°
- (f) Which of the following is a set of generators of $\mathbb{R}^3$
- $\{(4, 0, 0), (0, 0, 2)\}$
 - $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

Q. 3 Attempt the following (Any THREE) (15M)

- (a) Find the null space of matrix $\begin{bmatrix} 1 & 5 & 6 \\ 2 & 6 & 8 \\ 3 & 4 & 7 \end{bmatrix}$
- (b) Let $f: U \rightarrow V$ is a linear transformation then show that $\ker f = \{0\}$ iff f is injective.
- (c) Find the co-ordinate representation of vector $v = (0, 0, 0, 1)$ in terms of the vectors $[1, 1, 0, 1]$, $[0, 1, 0, 1]$ and $[1, 1, 0, 0]$ in $\text{GF}(2)$.
- (d) Find the angle between the two vectors $a = (2, 3, 4)$ and $b = (1, -4, 3)$ in $\mathbb{R}^3$.
- (e) Consider Subspace $U_1 \{(x, y, w, z) : x - y = 0\}$ and $U_2 \{(x, y, w, z) : x = w, y = z\}$ Find a basis and dimension of
- U_1
 - U_2
 - $U_1 \cap U_2$.
- (f) If V and W are two subsets of a vector space V such that U is a subset of W then show that W^0 is a subset of U^0 where U^0, W^0 are annihilator of U and W respectively.

Q. 4 Attempt the following (Any THREE) (15)

- (a) Let u and v are orthogonal vectors then prove that for scalars a, b .
- $$\|au + bv\|^2 = a^2\|u\|^2 + b^2\|v\|^2$$
- (b) Explain Internet Worm.
- (c) Write a program in python to find gcd (240, 24)

- (d) Solve the following system by Gaussian elimination method.
- $$\begin{aligned} y - z &= 3 \\ -2x + 4y - z &= 1 \\ -2x + 5y - 4z &= -2 \end{aligned}$$
- (e) Find the orthonormal basis for subspace $\mathbb{R}^4$ whose generators are $v_1 = (1, 1, 1, 1)$, $v_2 = (1, 2, 4, 5)$, $v_3 = (1, -3, -4, -2)$ Using Gram Schmidt orthogonalisation Method.
- (f) Let $a = (3, 0)$, $b = (2, 1)$ find vector in $\text{span}\{a\}$ that is closet to b is $b^{\parallel a}$ and distance $\|b^{\perp a}\|$.

Q. 5 Attempt the following (Any THREE)**(15)**

- (a) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear map defined by $f(x, y, z) = (x + 2y - z, x + y - 2z)$. Verify Rank T + Nullity $T = 3$.
- (b) Fill the table.

Vector space	Basis	Dimension
$\{0\}$		
$\mathbb{R}^2$	$\{(1, 0), (0, 1)\}$	
$P_2(x)$		3
$M_2(\mathbb{R})$		4
$\mathbb{R}$	$\{1\}$	

- (c) Find eigen values and eigen vectors of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$
- (d) Let S be a subset of vector space V . Prove that $S^\perp$ is a subspace of V .
- (e) Check whether the following set $\{(1, 1, 0), (0, 1, 1), (1, 1, 1)\}$ is linearly Independent or not.