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VCD/ SYCS SEM III Linear Algebra 75 MARKS $2\frac{1}{2}$HOURS

Note: (i) All questions are compulsory.
(ii) Figures to the right indicate marks .
(iii) Illustrations, in-depth answers and diagrams will be appreciated.
(iv) Mixing of sub-questions is not allowed.

Q.1) Attempt the following (Any Four) (20M)

a) Define Modulus and argument of complex number z. calculate modulus and argument for z=$2+2\sqrt{3}i$

b) Prove all five addition properties of vector space for the set $S = \{(x, 7x): x, y \in \mathbb{R}\}$

c) Explain the rotation of complex number. Also if z = 2+4i, then find coordinate of z in remaining three quadrant.

d) Solve the non-homogenous linear equations
$x + y + z = 3, x - 2y - 3z = 1, 2x + 3y + z = 9$

e) Find the square root of $z = 21 - 20i$

f) Define linear combination of vectors in vector space V. Verify (6,5,3) as a linear combination of (1,0,0), (1, 1,0) and (1,1,1).

Q.2) Attempt the following (Any Four) (20M)

a) Let $T: \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = x - y$. Show that T is a linear Transformation.

b) Find dimension of the vector space spanned by vectors (1,0,3), (0,4,0), (0,0,3), (2,1,3) and also find the basis.

c) Using Gauss elimination method solve the following equation
x + 5y = 7, -2x-7y = 5

d) Define linearly dependent and independent vectors. Verify S = {(1,0,0),(1,4,0), (1,4,6)} is linearly dependent or independent

e) Reduce the following matrix in a Row-Echelon form

$$\begin{bmatrix} 4 & 5 & 3 & 1 \\ 12 & 15 & 9 & 3 \\ 16 & 20 & 12 & 4 \\ 8 & 10 & 6 & 2 \end{bmatrix}$$

f) Define subspace of a vector space V.
Verify the set W= $\{(x, 0)/x \in IR\}$ is subspace of IR^2.

Q.3) Attempt the following (Any Four) (20M)

a) Find Eigen values and Eigen vectors for the matrix A=$\begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$

b) Use the Gram-Schmidt orthonormalization process to construct an orthonormal set of vectors from the linearly independent set {(1,0,2),(-1,0,1)}

c) Let X = (x_1, x_2) and Y= (y_1, y_2) be two vectors in IR^2 then prove that
$< X,Y > = x_1y_1 + 2x_2y_2$ is an inner product in IR^2.

d) Find minimal polynomial m(t) of $A = \begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & 4 \end{bmatrix}$

e) Check whether the set $\{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0), (0,0,1)\}$ is orthonormal.

f) Verify Cayley Hamilton Theorem for matrix $A=\begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$

Q.4) Attempt the following (Any Five) (15M)

a) Check whether the set {(1,2,1),(4,-2,0),(2,4,-10)} is orthogonal

b) Consider the basis $B = \{(2,1), (-1,3)\}$ *and* $B' = \{(1,0), (0,1)\}$. If u is a vector such that $u_B = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ then find $u_B{}'$

c) Verify the set $S=\{\begin{bmatrix} a & b \\ 1 & c \end{bmatrix} / a, b, c \in IR\}$ is a vector space in IR with respect addition and Scalar multiplication

d) Find distance and angle between two vectors x= (1,2,3) and y=(3,-2,1) in IR^3

e) Express the number $z=\frac{1+2i}{1-3i}$ in the polar form.

f) Find vector-matrix multiplication in terms of linear combination for $M=\begin{bmatrix} 1 & 2 & 4 \\ 10 & 15 & 20 \end{bmatrix}$ and v = (2, 4, 1)