Maths I Sem III Date. 21.9.2015

Date - 29/09/15

YCD - 5C75 MATHS I- SYBSC - SEM III ATKT EXAM - 75 MKS - $2\frac{1}{2}$ HRS

Note:: 1) All questions are compulsory.

2) For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks) from part (a), and any three subquestions (each 4 mks) from part (b)

3) For Q.4 Attempt any three.(each 5mks)

Q.1 (a) Attempt any one [Each 8]

1) Prove that if $x > 0, y > 0$ then i) $\exists\, n \epsilon IN$ such that $x < ny$ ii) $\exists\, n \epsilon IN$ such that $0 < \frac{1}{n} < y$ iii) $\exists\, n \epsilon IN$ such that $n - 1 \le x < n$.

2) State and prove Nested Interval Property for IR

Q.1 (b) Attempt any three. [Each 4]

1) Prove that if S is a set bounded above, it cannot have two suprema.

2) State and prove the Triangle inequality for IR

3) Prove that if $x \epsilon \mathbb{R}$ then there is $n \epsilon \mathbb{N}$ such that $x < n$.

4) Find least upper bound & greatest upper bound for S

where $S = \{ x \epsilon IR / x^2 - x - 6 < 0\}$

Q.2 (a) Attempt any one [Each 8]

1) State and prove Bolzano Weierstrass theorem of IR.

2) Define monotone increasing and monotone decreasing sequence. Also prove that every monotone increasing sequence converges to upper bound if bounded above.

Q.2 (b) Attempt any three. [Each 4]

1) When is a sequence (a_n) of IR ($n \epsilon IN$) said to be convergent? Using ϵ-δ definition, show that $(\frac{n+1}{n+2})$ is convergent and its limit is 1.

2) Define Limit Superior and Limit inferior and Lim sup and Lim inf of i) $\left\{\frac{1}{n}\right\}$ ii) $(-1)^n n$.

3) Define a Cauchy sequence in IR and prove that n^3 is not a Cauchy sequence in IR.

4) Examine whether following sequences are bounded or not?

i) $x_n = \frac{4}{n+2}$ ii) $a_n = 3n^2 + 6$.

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Q.3 (a) Attempt any one [Each 8]

1) State and prove Cauchy's criterion of convergence of series $\sum a_n$

2) Find fourier series of $f(x) = x^2$ in $[-\pi, \pi]$.

Q.3 (b) Attempt any three. [Each 4]

1) Let $\sum a_n, \sum b_n$ be convergent series converging to a,b respectively then if $\alpha \epsilon IR$ is fixed and $c_n = \alpha a_n$ then prove that $\sum c_n$ is convergent, converging to αa.

2) Determine convergence of series $\sum_{n=2}^{\infty} \frac{cos n\pi}{\sqrt{n}}$.

3) State and prove Modified ratio test for series $\sum_{n=1}^{\infty} a_n$.

4) State Cauchy's Root test and examine the convergence of $\sum \frac{4^n}{n^2}$.

Q.4 Attempt any three [Each 5]

1) Prove that If x & y are real numbers with x < y then there exists an irrational number z such that x< z <y.

2) State and prove that Hausdorff Property of IR.

3) State and prove Sandwich Theorem.

4) Examine whether following sequences are bounded or not?

i)$x_n = \frac{4}{n+2}$ ii)$a_n = 3n^2 + 6$.

5) Define the power series and the radius of convergence of power series and find the radius of convergence for $\sum \frac{1}{n3^n} x^n$.

6) State Comparison test for convergence of series and Examine the convergence of the series $\sum \frac{n}{n^2 - cos^2(n)}$.

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