VCD 25/10/23 **S.Y.B.Sc.(CS)-SEM-III-Linear Algebra-HRS-$2\frac{1}{2}$-MARKS-75**

Note: (i) All questions are compulsory.

(ii) Figures to the right indicate marks .

(iii) Illustrations, in-depth answers and diagrams will be appreciated.

(iv) Mixing of sub-questions is not allowed.

(v) Calculator is not allowed.

Q.1) Attempt the following (Any Four) **(20M)**

a) Define Modulus and argument of complex number z. calculate modulus and argument for $z=-1+i\sqrt{3}$

b) Find square root of complex number 7+24i.

c) Define addition, subtraction, multiplication and division of two complex number.

d) Define linear combination of vectors in vector space V. Express (2,4,9) as a linear combination of (1,1,0), (0,2,1) and (0,1,2).

e) Prove all five addition properties of vector space for the set $S=\left\{\begin{bmatrix} a & b \\ c & 0 \end{bmatrix} / \text{ a, b, c} \in \mathbb{R}\right\}$ of IR^2.

f) Solve the non-homogenous linear equations
$2x-3y+7z=5,\ 3x+y-3z=13,\ 2x+19y-47z=32$

Q.2) Attempt the following (Any Four) **(20M)**

a) Reduce the following matrix in a Row-Echelon form
$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & 2 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

b) Verify the subset S = {(1,2), (2,1), (1,1)} of $\mathbb{R}^2$is linearly Dependent.

c) Using Gauss elimination method solve the following equation
$$2x-y+3z=8\,,-x+2y+z=4, 3x+y-4z=0$$

d) Let $T:\mathbb{R}^2\to\mathbb{R}$ defined by $T(x,y)=x-y$. Show that T is a linear Transformation.

e) Prove that Every Superset of Linearly Dependent set is Linearly Dependent.

f) Define Basis of a vector space. Show that {(1,0),(1,1)} is basis of IR^2.

Q.3) Attempt the following (Any Four) **(20M)**

a) Find minimal polynomial m(t) of A =$\begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & 4 \end{bmatrix}$

b) Use the Gram-Schmidt orthonormalization process to construct an orthonormal set of vectors from the linearly independent set {(3,1),(4,2)}

c) Prove that $|<u,v>|\le\| u \|\| v \|$, for any two vectors in vector space V.

d) Verify Cayley Hamilton Theorem for matrix A=$\begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$

e) Let X = $(x_1, x_2, \dots . x_n)$ and Y= $(y_1, y_2, \dots \dots, y_n)$ be two vectors in IR^n then prove that $<X,Y> = x_1y_1 + x_2y_2 + \cdots \dots \dots + x_ny_n$ is an inner product in IR^n.

f) Find Eigen values and Eigen vectors for the matrix A=$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

Q.4) Attempt the following (Any Five) **(15M)**

a) Find distance and angle between two vectors x= (1,-2) and y=(-2,1) in IR^2

b) Consider the basis $B = \{(5,3),(3,2)\}$ *and* $B' = \{(1,0),(0,1)\}$. If u is a vector such that $u_B = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ Find $u_B{}'$

c) Define Dimension of a vector space with suitable example.

d) Explāin Galoi's field (GF_2), with addition and multiplication operation.

e) Express the number z= $\frac{3+2i}{(4+3i)(2+i)}$ in the form of a+ib.

f) Verify the set S={$(x, x^3)/ x \in IR$} is a vector Subspace in IR with respect addition and Scalar multiplication.