10/10/18

Q.P. Code: 31174

(2½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

**Q. 1 Attempt All (Each of 5Marks)**

**(a) Select correct answer from the following:** (15M)

1) In which of the following method, we approximate the curve of solution by the tangent in each interval.
a) Simpson's Method
b) Euler's method
c) Newton's method
d) None of the above

2) If $\int_0^a f(x)\,dx = 2\int_0^a f(x)\,dx$ then f(x) is called as
a) Even function
b) odd function
c) constant function
d) none of the above

3) A function is said to be invertible if and only if it is _____
a) Bijective b) injective c) Inflexion d) Surjective

4) $\lim_{x\to\infty} 1/3x =$
a) 1 b) infinite c) zero d) None

5) A point on a curve where two curves crosses each other is called_____.
a) Cusps b) Asymptote c) Tangent d) Point of inflexion

**(b) Fill in the blanks:**

*(continuous, ∞, −∞, Modelling, ¼, ½, 5x+5y-8, $a^x \log a$)*

1. $\lim_{x\to\infty}(-2x) =$ _____.
2. The derivative of $a^x$ is _____.
3. _____ is the process of writing a differential equation to describe a physical situation.
4. $\int_0^{\pi/2} \sin^3 x \cos x\,dx =$ _____
5. Linearization of $x^3 + xy + y^2$ at (1, 2) is _____.





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(c) Answer the following in one line

1. State $\in -\partial$ definition of limit
2. Define Concavity
3. Evaluate $\int [e^{2\log x} + e^{x\log a}]\ dx$
4. Define the term Definite Integral
5. Define Absolute Extreme values

Q. 2 Attempt the following (Any THREE) (15M)

(a) Show that $\lim_{x\to 3} \frac{x^3-4x^2+13x-30}{x-3} = 16$

(b) Show that $|x|$ is continuous everywhere.

(c) Find the intervals on which function $f(x)=x^2 - 4x + 3$ is increasing or decreasing.

(d) Find the relative extrema of $f(x)=-1+8x-3x^2$ using both first and second derivative test.

(e) Using Newton's method find the approximate root for the equation $f(x)=x^3 - x - 1$

(f) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 72 running feet of chicken wire is available for the fence?

Q. 3 Attempt the following (Any THREE) (15M)

(a) Evaluate $\int 1/(9\cos^2 x + 4\sin^2 x)\ dx$

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{(1+\cot x)}\, dx$

(c) Solve the differential equation $(1+y^2)\,dx + (1+x^2)\,dy = 0$

(d) Using Euler's method find approximate value of y when x = 1, in five steps, taking h = 0.2 for $dy/dx = x + y$ and $y(0) = 1$.

(e) Solve the differential equation $dy/dx + y \tan x = \cos^2 x$

(f) Evaluate $\int_1^2 \sqrt{1+x^2}\, dx$ using simpson's rule with n=6

Q. 4 Attempt the following (Any THREE) (15)

(a) Find $\lim_{(x,y)\to(4,-2)} x(y^3 + 2x)^{\frac{1}{3}}$

(b) Find the second order derivatives of $f(x,y)=y^2e^x + y$

(c) If $z=x^2+y^2$, $x = a\cos t$ and $y = a\sin t$. Use chain rule to find $\frac{dz}{dt}$.

(d) Find the directional derivative of $f(x, y, z)=x^2y - y z^3 + z$ at the point (1,-2,0) in the direction of the vector $a= 2i + j - 2k$

(e) Find the gradient vector of $f(x, y)$ if $f(x, y) = x^3 + 2xy^2$. Evaluate it at (-3, -4)

(f) Find the equation for the tangent plane and parametric equations for normal line to the surface $z=x^2y$ at the point (2, 1, 4)

**Q. 5 Attempt the following (Any THREE)** (15)

(a) Locate all relative extrema and saddle points of
$f(x, y)=x^3+2y^3-3x^2-24y+16$

(b) Solve the differential equation
$x(x+y)\,dy - y^2\,dx = 0$

(c) Sketch the graph of the equation $y= x^3+5x+7$ and identify the intervals where the function y is increasing and decreasing (draw the graph on the answer sheet itself).

(d) Evaluate $\int_0^{\pi/2} \sin 5x \cos 3x \, dx$

(e) Find the asymptotes of the function $y=\frac{x}{(x+1)(x+2)^2}$

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