

(2 ½ Hours)

[Total Marks: 75]

- N.B. 1) All questions are compulsory.  
 2) Figures to the right indicate marks.  
 3) Illustrations, in-depth answers and diagrams will be appreciated.  
 4) Mixing of sub-questions is not allowed.

**Q. 1 Attempt All (Each of 5Marks)****(15M)****(a) Select correct answer from the following:**

1) In which of the following method, we approximate the curve of solution by the tangent in each interval.

- a) Simpson's Method  
 b) Euler's method  
 c) Newton's method  
 d) None of the above

2)  $\int \frac{1}{(9x^2 + 25)} dx =$

- a)  $(3/5) \tan^{-1}(3x/5) + c$   
 b)  $(1/9) \tan^{-1}(3x/5) + c$   
 c)  $(3/5) \tan^{-1}(5x/3) + c$   
 d)  $(1/15) \tan^{-1}(3x/5) + c$

3) A function is said to be invertible if and only if it is \_\_\_\_\_

- a) Bijective      b) injective      c) Inflexion      d) Surjective

4)  $\lim_{x \rightarrow \infty} 7/2x =$

- a) 1      b) infinite      c) zero      d) None

5) If  $f(x, y) = x^3y^3 + y^3 + 1$  then  $f_x(x, y)$  is

- a)  $3x^2$       b)  $3xy$       c)  $y^3x$       d) None

**(b) Fill in the blanks:**

(continuous,  $\infty$ ,  $(4i+5j)/41$ ,  $(4i+5j)/31$ ,  $-\infty$ ,  $e^x$ , derivative,  $x - 3 \log|x+3| + c$ )

- $\lim_{x \rightarrow \infty} (5 - 2x) =$  \_\_\_\_\_.
- The derivative of  $e^x$  is \_\_\_\_\_.
- Unit vector of  $4i+5j$  is \_\_\_\_\_.
- $\int x/(x+3) dx =$  \_\_\_\_\_.
- The rate of change of one variable with respect to another is called \_\_\_\_\_.

(c) **Answer the following in one line**

1. Define Tangent Plane
2. Define Critical Point
3. Define the term Definite Integral

$$4. \text{ Evaluate } \int_{n/3}^{2\pi} \sin x \, dx$$

5. Linearization of a function

**Q. 2 Attempt the following (Any THREE)****(15M)**

- (a) Show that  $\lim_{x \rightarrow 1} 2x^2 + 3x - 4 = 1$
- (b) Discuss the continuity of the function  $f(x) = \sqrt{4 - x^2}$
- (c) Show that the function  $f(x) = x^3 - 9x^2 + 30x + 7$  is always increasing.
- (d) Find the relative extrema of  $f(x) = 4xy - x^4 - y^4$  using both first and second derivative test.
- (e) Using Newton's method find the approximate root for the equation  $f(x) = x - \cos x$
- (f) Divide 100 into two parts such that sum of their square is minimum.

**Q. 3 Attempt the following (Any THREE)****(15M)**

- (a) Evaluate  $\int \sin^{-1} \sqrt{x} \, dx$
- (b) Evaluate  $\int_{\pi/6}^{\pi/3} \frac{1}{(1 + \cot x)} \, dx$
- (c) Estimate  $\int_0^4 x^2 \, dx$  using Simpson's rule and  $n = 4$ .
- (d) Solve the differential equation  $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$
- (e) Solve  $dy/dx = 1 - y$ ;  $y(0) = 0$ , find  $y(0.1)$  and  $y(0.3)$  using Euler's method. Taking  $h = 0.1$ .
- (f) Solve the differential equation  $(x + 1) \frac{dy}{dx} - y = e^x (x + 1)^2$

**Q. 4 Attempt the following (Any THREE)****(15)**

- (a) Show that  $f(x, y) = 2x^2 + 3xy$  is continuous at  $(2, 3)$
- (b) Find the second order derivatives of  $f(x, y) = x^2 y^3 + x^4 y$
- (c) If  $z = x^2 y$ ,  $x = t^2$  and  $y = t^3$  Use chain rule to find  $\frac{dz}{dt}$ .
- (d) Find the directional derivative of  $f(x, y) = x^3 + 2xy^2$  at the point  $(-2, -3)$  in the direction of the vector  $a = i + j$
- (e) Find the gradient vector of  $f(x, y)$  if  $f(x, y) = 10 - 8x^2 - 2y^2$ . Evaluate it at  $(2, 3)$
- (f) Find the equation for the tangent plane and parametric equations for normal line to the surface  $z = x^2 y$  at the point  $(2, 1, 4)$

**Q. 5 Attempt the following (Any THREE)****(15)**

- (a) Locate all relative extrema and saddle points of

$$f(x, y) = 3x^2 - 2xy + y^2 - 8y$$

- (b) Solve the differential equation

$$\frac{dy}{dx} = (4x + y + 1)^2$$

- (c) Draw the graph of
- $y = 4 - 3x^2 + x^3$
- and find the intervals on which the function
- $y$
- is increasing and decreasing (draw the graph on the answer sheet itself)

- (d) Find the asymptotes of the function
- $y = \frac{x}{(x+1)(x+2)^2}$

- (e) Solve the differential equation

$$dy/dx = (4x + y + 1)^2$$

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