(Time: 2½ Hours) **[Total Marks: 75]**

N.B.
1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Answer the following questions **(15M)**

(a) Choose the best choice for the following questions: **(5M)**

(i) A function f from R to R which satisfies f(a) = f(b) implies a=b for every a and b in R is said to be
(a) One-to-one function (b) onto function
(c) Either one-to-one or onto function (d) None of these

(ii) A relation R on a set X is such that whenever (x, y) ∈ R, (y, x) ∈ R, then R is called
(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

(iii) What is the coefficient of $x^2 y^2$ in the expansion of $(x+y)^4$:
(a) 4 (b) 6 (c) 8 (d) None of these

(iv) Suppose a bookcase shelf has 5 Physics texts, 3 Chemistry texts, 6 Biology texts, and 4 Mathematics texts. Number of ways a student can choose one text of each type is given by
(a) 660 (b) 560 (c) 460 (d) None of these

(v) An undirected graph with no multiple edges or loops is called
(a) Simple graph (b) Complex graph (c) Tree (d) Pseudo graph.

(b) Fill in the blanks for the following questions: **(5M)**

(i) A function f such that f(x) = x for any x in the domain of f is said to be a _______________function.

(ii) A relation R on a set A is called _________ if whenever (a, b) ∈ R, then (b, a) ∈ R, for all a, b ∈ A.

(iii) The Gödel number of a word $w = a_5a_2a_3a_1a_2$ is________

(iv) The number of different license plates that can be made if each plate contains a sequence of three uppercase English letters followed by three digits is given by ___________.

(v) Let G be a directed graph and v be a vertex of G. The number of edges ending at v is called ___________.

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(c) Answer the following questions: **(5M)**

- **(i)** If the domain of the function f(x) = x+1 is R, what will be its co-domain?
- **(ii)** Let S be a set. Determine whether there is a greatest element and a least element in the poset (P(S), ⊆).
- **(iii)** How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?
- **(iv)** Define a regular grammar.
- **(v)** What is the degree of a vertex of n undirected graph?

Q. 2 Answer any *three* of the following: **(15M)**

- **(a)** Determine whether the function f: R-> R given by f(x) = -3x + 4 is a bijection.
- **(b)** Find the domain and range of following functions:
  - (i) The function that assigns to each positive integer the number of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 that do not appear as decimal digits of the integer.
  - (ii) The function that assigns to a bit string the numerical positionofthefirst1inthestringandthatassignsthe value 0 to a bit string consisting of all 0s
- **(c)** Draw the Hasse diagram representing the partial ordering {(a,b) / a divides b} on {1,2,3,4,6,8,12}.
- **(d)** Which of these relations on {0, 1, 2, 3} are partial orderings?
  - (i) {(0,0),(2,2),(3,3)}
  - (ii) {(0,0),(1,1),(2,0),(2,2),(2,3),(3,3)}
- **(e)** Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0.
- **(f)** Find the solution of the recurrence relation $a_n = a_{n-1} + 2a_n - 2$ with $a_0 = 2$ and $a_1 = 7$.

Q. 3 Answer any *three* of the following: **(15M)**

- **(a)** How many permutations of the letters ABCDEFG contain:
  - (i) The string BCD?
  - (ii) The string CFGA?
  - (iii) The strings BA and GF?
  - (iv) The strings ABC and DE?
  - (v) The strings ABC and CDE?
- **(b)** State and prove Pascal identity.
- **(c)** State Pigeonhole principle. A chess player has 77 days to prepare for an important tournament. He decides to practice by playing at least one game per day and a total of 132 games. Show that there is a succession of days during which he must have played exactly 21 games.
- **(d)** Suppose that there are nine students in a discrete mathematics class at a small college.
  (i) Show that the class must have at least five male students or at least five female students.
  (ii) Show that the class must have at least three male students or at least seven female students.

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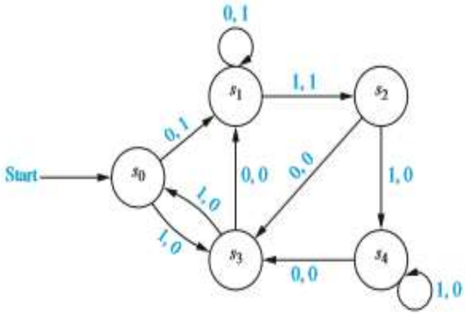
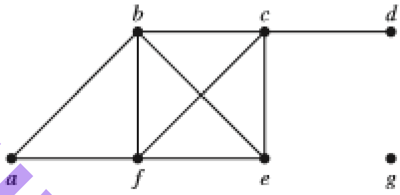
(e) Construct a derivation tree for the following derivation:
the hungry rabbit eats quickly.

(f) Find the output string generated by the finite-state machine given below if the input string is 101011.



Q. 4 **Answer any *three* of the following:** **(15M)**

(a) Find the degree and neighborhood of each of the vertex of the graph given below:



(b) Suppose a graph G contains two distinct paths from a vertex u to a vertex v. Show that G has a cycle.

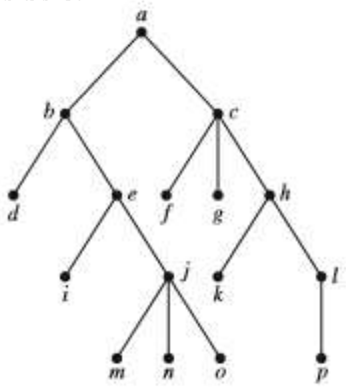
(c) Draw the graph corresponding to the following adjacency matrix:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

(d) Represent the following expressions using binary tree:
(i) (x +xy)+(x/y); (ii) x +((xy +x)/y).

(e) Draw all possible non similar binary trees T with four external nodes.

(f) Determine the order in which a preorder traversal visits the vertices of the following ordered rooted tree:



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Q. 5 Answer any *three* of the following: **(15M)**

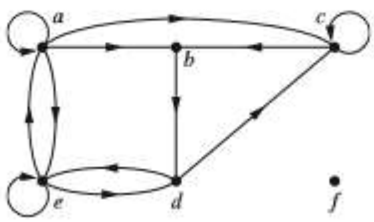
(a) Let R be the relation on the set of all people who have visited a particular Web page such that xRy if and only if person x and person y have followed the same set of links starting at a particular Web page. Show that R is an equivalence relation.

(b) Find the solution of the recurrence relation $a_n = 6a_{n-1} - 9a_n - 2$ with initial conditions $a_0 = 1$ and $a_1 = 6$.

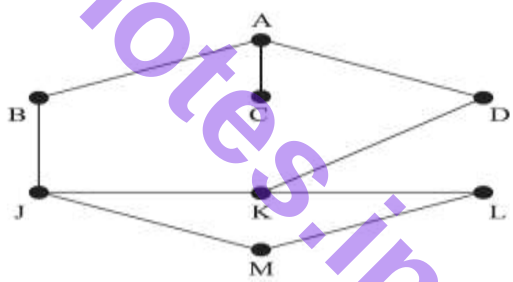
(c) What is the coefficient of $a^{13}b^{123}$ in the expansion $(a+b)^{25}$ using binomial theorem.

(d) Define a language L over an alphabet A. Let A= {a, b, c}. Find L* where language L= {b2}.

(e) Find the in-degree and out-degree of each vertex in the graph shown:



(f) Consider the graph *G* in the following figure (where the vertices are ordered alphabetically). (i) Find the adjacency structure of *G*. (ii) Find the order in which the vertices of *G* are processed using a Breadth-first search algorithm beginning at vertex *A*.



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