9/12/23

VCD/ FYCS SEM I Discrete Mathematics 75 MARKS $2\frac{1}{2}$ HRS

1. **All** questions are **compulsory**.
2. **All questions** carry **equal** marks.
3. Draw **neat, labeled diagrams** wherever necessary.

1. Attempt the following (Any four) **(20 M)**

a. Prove that the function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \frac{2x-3}{7}$ is a bijective. Hence find its inverse.

b. Let $f(x) = x^3, g(x) = x - 5$ are the functions Find i) fog and gof
ii) $fog(-2)$ and $gof(4)$

c. Let $A = \{1,2,3\}, B = \{a,b,c\}$ and $C = \{x,y,z\}$. Consider the following relations R and S from A to B and B to C respectively.
$R = \{(1,b),(2,a),(2,c)\}$ and $S = \{(a,y),(b,x),(c,y),(c,z)\}$.
Find SoR, M_R, M_S and M_{SoR}

d. Let R be the relation on the set $A = \{2,4,8,16,32\}$ where $R = \{(a,b): a \mid b\}$
Draw the Hasse diagram

e. Solve the following linear homogeneous recurrence relation
$a_n = 3a_{n-1} + 4\,a_{n-2}$, $a_0 = 1, a_1 = 1, n \geq 2$

f. Let $a_n = 2^n + 5(3^n)$ for $n = 0,1,2 \ldots$
Show that i) $a_2 = 5a_1 - 6\,a_0$ ii) $a_3 = 5a_2 - 6\,a_1$

2. Attempt the following (Any four) **(20 M)**

a. How many 4-digit codes can be formed using the digits 0 – 9 if
i) repetition of digit is not allowed
ii) repetition of digit is allowed

b. How many positive integers not exceeding 100 are divisible either by 4 or by 6?

c. In an urn contains 15 balls out of which 8 are white and 7 are black. In how many ways can 5 balls be selected so that atmost 3 are black?

d. Prove that $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

e. What is the coefficient of $x^{12}y^5z^{13}$ in the expansion $(x+y+z)^{30}$

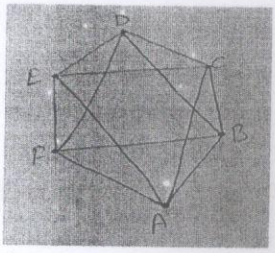
f. Let M be the FSM defined by the following state table:

F	A	B
$\to s_0$	s_1, x	s_2, y
s_1	s_3, y	s_1, z
s_2	s_1, z	s_0, x
s_3	s_0, z	s_2, x

Find i) states ii) input letters iii) output letters iv) initial state
v) $f(s_3, a)$ vi) draw the state diagram

3. Attempt the following (Any four) **(20 M)**

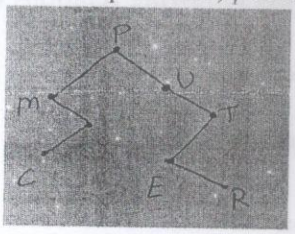
a. Write the adjacency structure for the following graph:



b. Construct the tree from the algebraic expression:

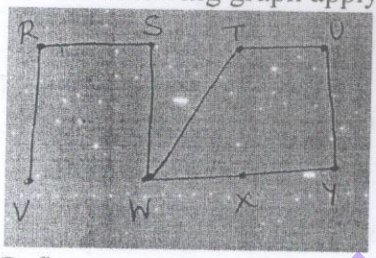
$$(((a \times b) - c) \wedge d) - ((e \times f) + g)$$

c. Perform preorder, postorder and inorder search for the following tree:



d. Define path, cycle, trail with a suitable example.

e. For the following graph apply BFS taking S as starting vertex



f. Define terms related to graph

i) adjacent vertex ii) degree of a vertex iii) pendent vertex

iv) loop v) incidence

4. Attempt the following (Any five) **(15 M)**

a. Define partial order set and transitive.

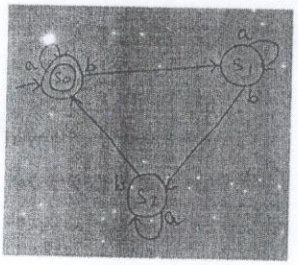
b. Show that $a_n = 1$ is not a solution the of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$

c. Define sum and product rule.

d. In how many arrangement of the word LETTER contains the two T's together?

e. Form a binary search tree for the following: The, hungry, rabbit, eats, quickly

f. Consider the FSA defined by the state diagram. Find its state table.



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